

2-D inversion of dipole-dipole and shallow electromagnetic data by Internet

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ABSTRACT

The purpose of this two program codes is to obtain a resistivity image of a 2-D heterogeneous subsurface by mean of inversion of dipole-dipole (D-D) data or shallow electromagnetics (as EM-34, EM-31). Both methods can be very useful for detecting lateral and vertical variations of the ground resistivity. We pretend that many people use these programs and increase the popularity of the dipole-dipole over the Schlumberger array and also of the EM-34. Thanks to the present level in internet technology it is possible to build a WEB page where anybody around the world can access freely and introduce their data file in order to get a resistivity picture of the subsurface.

INTRODUCTION

The Schlumberger array have been very popular maybe because there are many forward and inversion techniques for the interpretations in terms of layers (1-D). However dipole-dipole is not as popular despite is sensible to both lateral and vertical variations of the resistivity. The EM-34 is an electromagnetic device and consist of two loops, one is the transmitter and another the receiver. The transmitter circulates a current with three different frequencies, 600 Hz for intercoil distance of 10 m, 1600 Hz for 20 m and 6400 Hz for 40 m. The transmitter induces a magnetic field in the earth that interact with the conductors in it. In the receiver arrives a second magnetic field due to the conductors plus the primary field from the source. The rate of both fields is the measurement and it is a complex function. For low frequencies the quadratic component is a straight line with a slope that depends of the average conductivity in the zone of influence. The real part does not depend of the ground conductivity only of the distance between transmitter and receiver. This value is used by the equipment to advice the surveyor when this reached the programmed distance of 10, 20 or 40 meters. This equipment was built by Geonics Co. mainly for mine purposes. The company provides a program code for a layered earth (1-D) when using the three separations in horizontal and vertical loops. However this equipment is sensible also to lateral variations of the conductivity. At the moment of the writing of this paper there are not other program codes developed for inversion of this data assuming a 2-D structure in subsurface. We developed an individual inversion technique for dipole-dipole and EM-34, EM-31 data assuming 2-D structures (Pérez-Flores et al. (2001)). The idea in this paper is to show to any geophysical user to manipulate their field data and to input it in a web page where the program will run in our server without any cost. The output files are a section of the resistivity of the subsurface, a file with the model response in order to compare with the data and a misfit.

METHOD DESCRIPTION

As the reader can check in Pérez-Flores et al. (2001), the program does not use scattering theory to express our integral equation. We use the theory developed by Gómez-Treviño (1987). From that theory we can arrive to the equations written in Pérez-Flores et al. (2001) that express that any dipole-dipole or EM-34 resistivity measurement is a weighted average of the resistivity distribution in the earth. The inversion process is a kind of de-averaging of measurements in order to estimate the resistivity or conductivity of a conglomerate of 2-D prisms that shape a plane half-space as we can see in figure 1. The equation used for dipole-dipole has the form,

$$\log \mathbf{r}_a(x_A, x_B, x_M, x_N) = \int_{x_1}^{x_2} \int_{z_1}^{z_2} K_e(x_A, x_B, x_M, x_N, x, z) \log \mathbf{r}(x, z) \, dx dz . \quad 1$$

Where the apparent resistivity (\mathbf{r}_a) measured depends of the location in x of the source (A , B) and of the receiver (M , N). The kernel or weighting function (K_e) depends of the location and characteristics of source and receiver and the subsurface grid that represents the conglomerate of 2-D prisms shaping the earth half space. This function depends of the characteristics of the grid but not of the resistivities in the prisms. The electrical property to be estimated by the inversion process is $\mathbf{r}(x, z)$ in every 2-D prism (figure 1).

The integral representation does not involve y coordinate because this integral was done analytically from $-\infty$ to ∞ because the resistivity is assumed constant in that direction.

The integrals over x and z must be done numerically because we do not know the geometry of the heterogeneities in the subsurface.

If we assume a constant resistivity half-space, the apparent resistivity must be the same of the half-space. This is possible only if the double integral of the kernel function is unity for every measurement location. This is equivalent to take off the prisms resistivity from the integrals. This is very useful to control the numerical integration precision.

The equation for shallow electromagnetics is,

$$\mathbf{s}_a(x_s, x_r) = \int_{x_1}^{x_2} \int_{z_1}^{z_2} K_m(x_s, x_r, x, z) \mathbf{s}(x, z) \, dx dz . \quad 2$$

Where \mathbf{s}_a represents the apparent conductivity and depends of x location of the source and receiver. For horizontal loops \mathbf{s}_a can be negative close to a good and surface conductor.

The kernel function (K_m) depends of the magnetic fields from the source, receiver and the subsurface grid of prisms. $\mathbf{s}(x, z)$ is the prism conductivity to be estimated, notice that the kernel does not depend of these. Here also the integral of the kernel must be unity.

The dipole-dipole program runs with the logarithm of the resistivity and the EM-34 with the conductivity. As we can see in equation 2, for EM-34 the conductors are more important than resistives. In equation 1 for D-D, conductors and resistives are equally important thanks to the logarithms.

For the inversion process we use quadratic programming that allow us to put constraints over the resistivities or conductivities of the prisms. The objective function for the DC resistivity data is,

$$F(\tilde{\mathbf{n}}) = \left\| \tilde{\mathbf{n}}_a - \mathbf{K}_e \tilde{\mathbf{n}} \right\|_{Cy}^2 + \mathbf{b} \left\| \mathbf{D}\tilde{\mathbf{n}} \right\|^2 . \quad 3$$

Where $\tilde{\mathbf{n}}_a$ is a column matrix that represents the apparent resistivity logarithm, the size is the number of measurements, $\tilde{\mathbf{n}}$ is another column matrix that represents the resistivity logarithm of the prisms, the size is the number of prisms that shape the subsurface (the unknowns). \mathbf{K}_e is the weighting function matrix, it has as many rows as the number of measurements and as many columns as the number of prisms. \mathbf{D} is a spatial differential operator, $\mathbf{D}\tilde{\mathbf{n}}$ contains the derivatives of the resistivity logarithm with respect to x and z . \mathbf{b} (smoothing factor) is a scalar that pretends to equalize the importance of the second term of equation 3 with respect to the first term. The first term obligates to the inversion process to fit the data and the second term to find the smoother model. This means that the difference between the resistivities of two prisms in the x direction must not be great, the same happens in the z direction. This is very useful in the further prism from surface where the data has not enough resolution. The second term allows just the resistivity jumps needed to fit the data and avoid unnecessary roughness in the model. This is the principle of ‘‘Occam knife’’.

An equation similar to equation 2 is used to the shallow electromagnetic data, but substituting $\tilde{\mathbf{n}}$ by \mathbf{o} .

When we increase the smoothing factor we get a smoother resistivity model and we increase the misfit. When we decrease the smoothing factor we get a roughness resistivity model and decrease the misfit. A compromise between the two terms of equation 2 is needed in order to do not over fit the data and get unnecessary roughness.

The program code consumes 90% of the computer time in calculating the weighting function and 10% in the inversion process to get the prisms resistivities or conductivities.

The total time depends of the number of measurements, unknowns and the CPU velocity of the computer. But a small section consumes around 10 min in a computer of 166 MHz.

Because the weighting function does not depends of the prism resistivities, only of the geometry of the grid and measurements locations (see equations 1 and 2), we can compute this matrix once in a projected dipole-dipole or EM-34 line. Then we can get as many models (changing the smoothing factor) in just 10% of the total time and to choose the model with the best geological sense.

PROGRAM DESCRIPTION

The input file must contain information about the geometry of the prisms grid, the number of measurements, the measurements, the smoothing parameter and some data about the output plot. We have not constructed an automatic grid because this must depend of the geophysicist criteria.

The harder part is how to built this grid. The better way is by mean of an example. For such a case we will use the input file showed in Table 1 and as the output plot that showed in figure 2. The description of table 1 is as follows:

- The row number 1: First term, the number of sets of prisms of the same dimensions. As we see in figure 1, the grid consist of a finer grid where the observation where taken (survey area) and a set of two broad blocks at the left side and another set to the right. These sided sets of blocks simulate that the earth extents far enough from the zone of influence of the electrode position. We assume that every prism will have a constant resistivity and the inversion code will find the resistivity for every prism. In our example, the first set consist of two blocks with a wide of 200 m in x direction with the beginning at $x=-445\text{ m}$. The second set consists of 42 prisms with a prism wide of 5 m and beginning in $x=-45\text{ m}$. The set number three is equal to set 1 but beginning in 165 m . For this example we define three sets of prisms for every layer (see figure 1). First layer goes from $z=0$ to $z=2\text{ m}$, layer two $z: (2, 5)$, layer three $z: (5, 10)$, layer four $z: (10, 18)$, layer five $z: (18, 30)$, layer six $z: (30, 50)$, layer seven $z: (50, 100)$, layer eight $z: (100, 200)$, layer nine $z: (200, 400)$ as we can see in figure 1. We recommend a logarithmic criterion in the z -thick of every prisms layer, because the resolution of the data dismisses with the depth. If we have three sets per layer and 9 layers we have a total of 27 sets that is the first number of input file. The second number of row one is the number of dipole-dipole measurement. For the EM-34 program it ask first for the number of vertical loop measurements and then the number of horizontal loop (table 2).
- Row number 2 to 4: Contains the x beginnings of every set. In our example every layer has three sets with 9 layers. The data can be input in columns or rows as in this example.
- Row number 5 to 7: Contains the z beginning of every set. Notice that we used a logarithmic criterion.
- Row number 8 to 10: Contains the x wide of every prism into their corresponding set.
- Row number 11 to 13: The z wide of every prism in their corresponding set.
- Row number 14 to 16: The number of equal prisms in x direction of every set.
- Row number 17 to 19: The number of equal prisms in z direction for every set. In our example we use a logarithmic criterion but if we use a lineal criterion with layer of same wide, we can save many sets. Maybe with three sets would be enough. It would be less tedious but less realistic.
- Row number 20: First term, the smoother parameter, we suggest 10^{-3} , 10^{-2} and 10^{-3} . Term two, the error in percent units. The error will be applied to all the data. We suggest 0 to 5%. Term three and four, are the solution bounds for all the variables (from quadratic programming). In this case we ask for a model between 1 and 1000 ohm-m . The resistivity can not be 0 because of the logarithm (equation 1). Term five, for the first run must be 0 in order to compute the weighting

function. When computed it is written in the hard disk in binary way. For the second run with another smoothing parameter we can change this term to l in order to read the binary matrix from the disk and save 90% of the computer time.

- Row number 21: It has the plot specifications. The program generates two postscript files, one in color and another in gray tones. Term one and two, are the beginning and end of the plot in x direction. Term three and four are the beginning and end in z direction. Terms five and six are the number of tics in x and z direction respectively. Terms seven and eight are the first and last mark in the color or gray scale. This scale is a histogram with seven colors or gray tones. Term nine, if 0 shows the resistivity, if l shows the logarithm of the resistivity (see figure 2).
- Row number 22: It is a maximum 50-character title.
- Rows number 23 to 121: These are the measurements. Term one, is the center of the four electrodes array in x direction. Term two, is " n ", multiplied by " a " is the distance between source and receiver dipoles and it can be an entire or decimal number. Term three is the electrodes separation in the source or the receiver " a ". Notice that this can change along the line. Term four is the apparent resistivity in $ohm-m$. The program code computes the logarithm internally (equation 1). For example in row 23, the center is in $x=-30 m$. If $a=10 m$, and $n=1$, the four electrode locations are $x: (-45, -35, -25, -15) m$. Remember that reciprocity is applied in dipole-dipole and EM-34, so it does not matter if we confuse the source with the receiver location. All the measurements must be co-linear. We assume a flat surface, so the resulting model can contain topographic effects mainly in the first row of prisms.
- Rows number 122 to 124: Contains information for the quadratic programming code. These are the *start*, *end* as well as the kind of minimization. In our case we minimize the quadratic norm (see equation 3).

If the user type all the Table 1 data will obtain the same picture as in figure 2a. Also he will obtain a color picture that is visually more informative than this. If the user type Table 2 will obtain the picture showed in figure 2b. Also the user will obtain an output file with the resistivity or conductivity and the coordinates of the center of every prism, the measurements contrasting with the response and a general RMS error. Also you obtain a file that inform if the minimization was optimal. Optimal means that the minimum was found between the input bounds. If the message is not optimal maybe we need to open the input bounds. Sometimes an error file appears when the numerical integration is not close to unity. This can be fixed if we use smaller prisms in the survey area (figure 1). If the blocky pictures are not enough for the user it is easy to construct a better interpolated picture by mean of the corresponding output file. In our case we used *Surfer* to get smoother and more beautiful images as shown in figures 4a and 4b.

CONCLUSIONS

Thanks to the advance of Internet tools we can offer new techniques of geophysical interpretations to geophysicist or geologist whose main goal is to apply and not to develop the techniques.

Dipole-dipole inversion in 2-D has proved been very useful for water table determination, saline intrusions and contaminants movement in the subsurface.

The electromagnetic device EM-34 despite is less resolutive because the equipment is limited to only three separations between source and receiver. But has the advantage that collecting the data is much faster than dipole-dipole and also is very useful for water table determination, contaminants and archeology.

There are emerging some new electrical equipments like the *Ohmeter* of *Geometric Co.* that allow us to collect the data faster than EM-34 and it is possible to use the same software proposed in this paper.

REFERENCES

- GÓMEZ-TREVIÑO, E., 1987. Nonlinear integral equations for electromagnetic inverse problems. *Geophysics*, **52**, 1297-1302.
- PÉREZ-FLORES, M.A., S. MÉNDEZ-DELGADO, and E. GÓMEZ-TREVIÑO, 2001, Imaging low-frequency and DC electromagnetic fields using a simple linear approximation. *Geophysics*, **66**, 1067-1081.

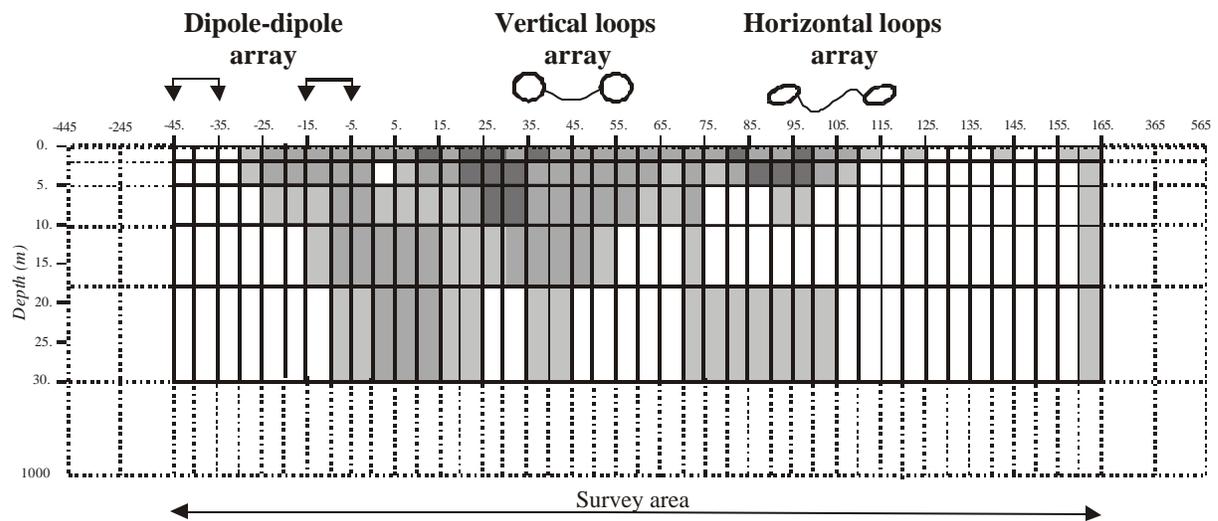
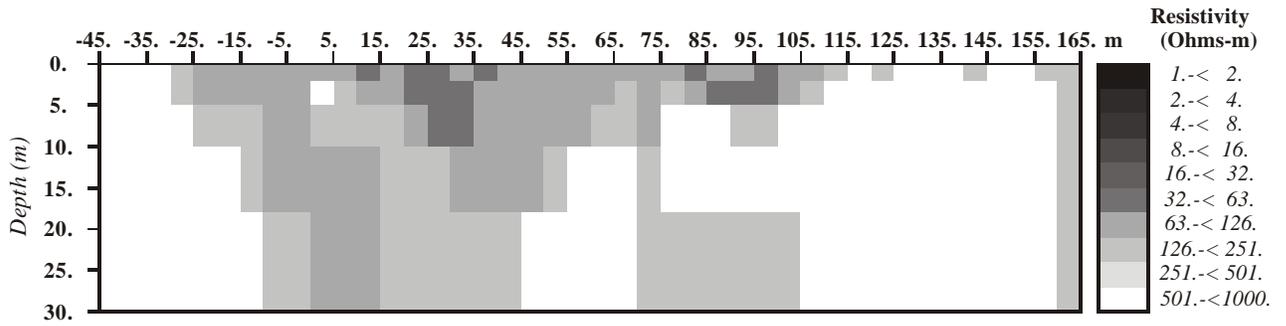


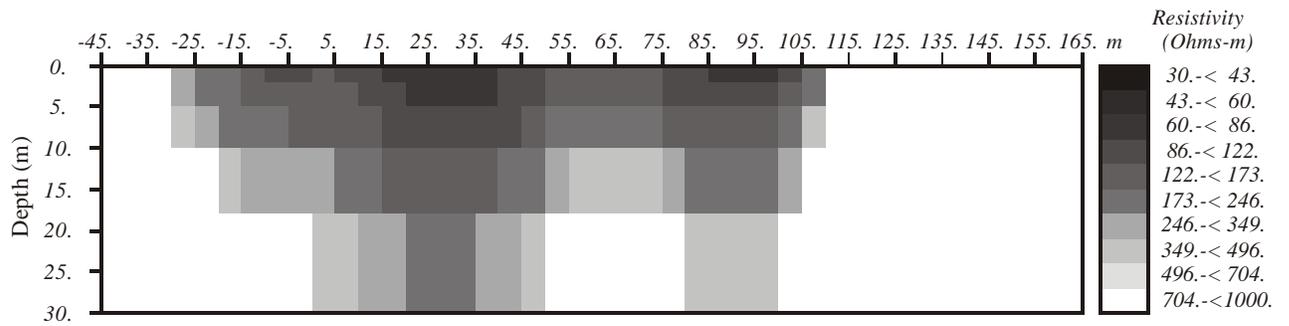
Fig. 1. The subsurface consists of grid finer in survey region and broad at the sides and bottom. The goal of the inverse programs is to estimate the resistivity or conductivity of every 2-D prismatic cell.

DC Dipole-data from LAS AURAS



(a)

EM-34 data from LAS AURAS



(b)

Fig. 2. These are the postscript files generated by the program code. These are resistivity images as shown by the color scale. (a) It is the image for the dipole-dipole data. (b) it is the image for the EM-34 data.

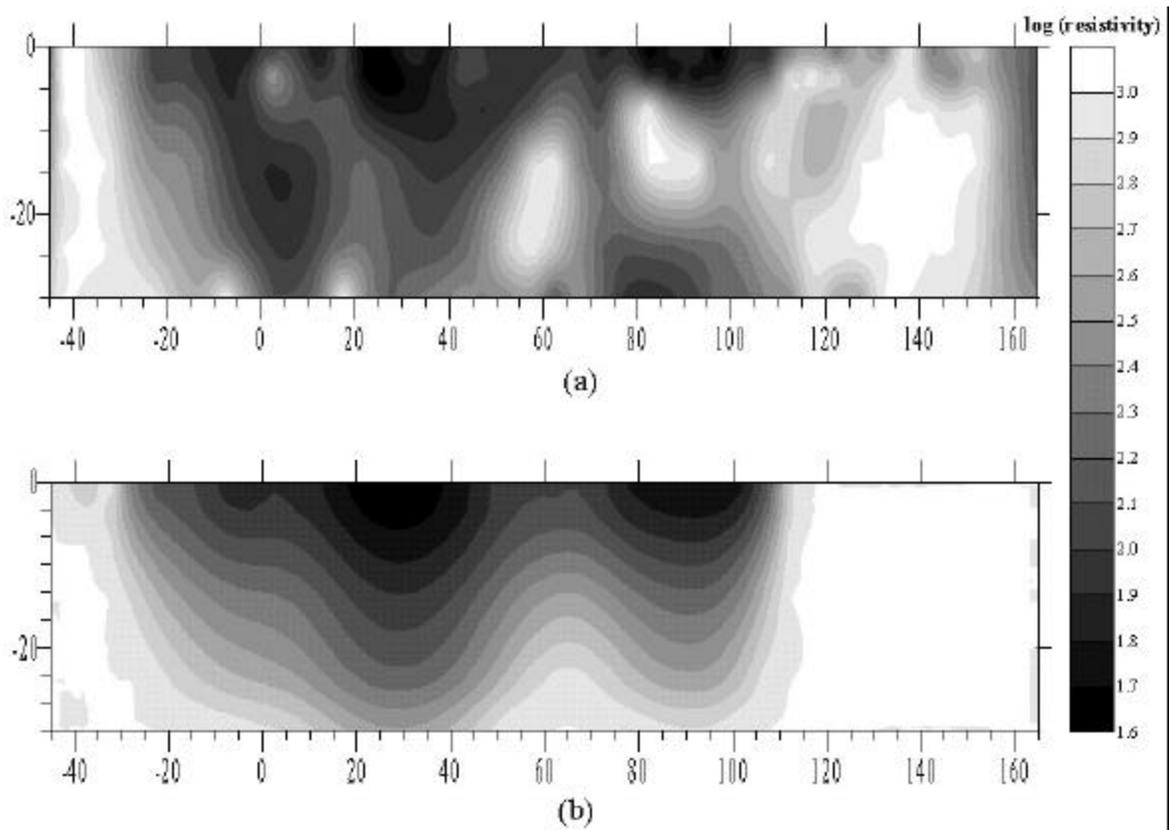


Fig. 3. Same resistivity images shown in figure 2 but in surfer using the the output file sended by the program code. (a) Resistivity image for dipole-dipole data. (b) For EM-34 data.

Table 1

INPUT FILE: DIPOLE-DIPOLE									
1:	27	99							
2:	-445.00	-45.00	165.00	-445.00	-45.00	165.00	-445.00	-45.00	165.00
3:	-445.00	-45.00	165.00	-445.00	-45.00	165.00	-445.00	-45.00	165.00
4:	-445.00	-45.00	165.00	-445.00	-45.00	165.00	-445.00	-45.00	165.00
5:	0.00	0.00	0.00	2.00	2.00	2.00	5.00	5.00	5.00
6:	10.00	10.00	10.00	18.00	18.00	18.00	30.00	30.00	30.00
7:	50.00	50.00	50.00	100.00	100.00	100.00	200.00	200.00	200.00
8:	200.00	5.00	200.00	200.00	5.00	200.00	200.00	5.00	200.00
9:	200.00	5.00	200.00	200.00	5.00	200.00	200.00	5.00	200.00
10:	200.00	5.00	200.00	200.00	5.00	200.00	200.00	5.00	200.00
11:	2.00	2.00	2.00	3.00	3.00	3.00	5.00	5.00	5.00
12:	8.00	8.00	8.00	12.00	12.00	12.00	20.00	20.00	20.00
13:	50.00	50.00	50.00	100.00	100.00	100.00	200.00	200.00	200.00
14:	2	42	2	2	42	2	2	42	2
15:	2	42	2	2	42	2	2	42	2
16:	2	42	2	2	42	2	2	42	2
17:	1	1	1	1	1	1	1	1	1
18:	1	1	1	1	1	1	1	1	1
19:	1	1	1	1	1	1	1	1	1
20:	1.0e-3	0.0	1.0	1000.0	1				
21:	-45.0	165.0	0.0	30.0	22	7	1.0	1000.0	1
22:	'DC Dipole-data from LAS AURAS'								
23:	-30.0	1.0	10.0	265.0					
24:	-20.0	1.0	10.0	148.0					
25:	-10.0	1.0	10.0	101.0					
26:	0.0	1.0	10.0	99.0					
27:	10.0	1.0	10.0	143.0					
28:	20.0	1.0	10.0	64.0					
29:	30.0	1.0	10.0	52.0					
30:	40.0	1.0	10.0	79.0					
31:	50.0	1.0	10.0	93.0					
32:	60.0	1.0	10.0	132.0					
33:	70.0	1.0	10.0	118.0					
34:	80.0	1.0	10.0	125.0					
35:	90.0	1.0	10.0	62.0					
36:	100.0	1.0	10.0	133.0					
37:	110.0	1.0	10.0	238.0					
38:	120.0	1.0	10.0	631.0					
39:	130.0	1.0	10.0	729.0					
40:	140.0	1.0	10.0	612.0					
41:	150.0	1.0	10.0	307.0					
42:	-25.0	2.0	10.0	248.0					
43:	-15.0	2.0	10.0	174.0					
44:	-5.0	2.0	10.0	124.0					
45:	5.0	2.0	10.0	94.0					
46:	15.0	2.0	10.0	86.0					
47:	25.0	2.0	10.0	86.0					
48:	35.0	2.0	10.0	75.0					
49:	45.0	2.0	10.0	75.0					
50:	55.0	2.0	10.0	128.0					
51:	65.0	2.0	10.0	147.0					
52:	75.0	2.0	10.0	160.0					
53:	85.0	2.0	10.0	132.0					
54:	95.0	2.0	10.0	187.0					
55:	105.0	2.0	10.0	339.0					
56:	115.0	2.0	10.0	206.0					
57:	125.0	2.0	10.0	1108.0					
58:	135.0	2.0	10.0	1066.0					
59:	145.0	2.0	10.0	528.0					
60:	-20.0	3.0	10.0	308.0					
61:	-10.0	3.0	10.0	233.0					
62:	0.0	3.0	10.0	134.0					
63:	10.0	3.0	10.0	59.0					
64:	20.0	3.0	10.0	110.0					
65:	30.0	3.0	10.0	123.0					
66:	40.0	3.0	10.0	89.0					
67:	50.0	3.0	10.0	109.0					
68:	60.0	3.0	10.0	131.0					
69:	70.0	3.0	10.0	170.0					
70:	80.0	3.0	10.0	136.0					
71:	90.0	3.0	10.0	357.0					
72:	100.0	3.0	10.0	401.0					
73:	110.0	3.0	10.0	286.0					
74:	120.0	3.0	10.0	289.0					
75:	130.0	3.0	10.0	1200.0					
76:	140.0	3.0	10.0	716.0					
77:	-15.0	4.0	10.0	390.0					
78:	-5.0	4.0	10.0	221.0					
79:	5.0	4.0	10.0	85.0					
80:	15.0	4.0	10.0	75.0					
81:	25.0	4.0	10.0	145.0					
82:	35.0	4.0	10.0	142.0					
83:	45.0	4.0	10.0	137.0					
84:	55.0	4.0	10.0	100.0					
85:	65.0	4.0	10.0	141.0					
86:	75.0	4.0	10.0	135.0					
87:	85.0	4.0	10.0	346.0					
88:	95.0	4.0	10.0	670.0					
89:	105.0	4.0	10.0	332.0					
90:	115.0	4.0	10.0	339.0					
91:	125.0	4.0	10.0	295.0					
92:	135.0	4.0	10.0	732.0					
93:	-10.0	5.0	10.0	343.0					
94:	0.0	5.0	10.0	139.0					
95:	10.0	5.0	10.0	109.0					
96:	20.0	5.0	10.0	107.0					
97:	30.0	5.0	10.0	157.0					
98:	40.0	5.0	10.0	211.0					
99:	50.0	5.0	10.0	120.0					
100:	60.0	5.0	10.0	107.0					
101:	70.0	5.0	10.0	105.0					
102:	80.0	5.0	10.0	342.0					
103:	90.0	5.0	10.0	661.0					
104:	100.0	5.0	10.0	585.0					
105:	110.0	5.0	10.0	425.0					
106:	120.0	5.0	10.0	384.0					
107:	130.0	5.0	10.0	189.0					
108:	-5.0	6.0	10.0	211.0					
109:	5.0	6.0	10.0	190.0					
110:	15.0	6.0	10.0	148.0					
111:	25.0	6.0	10.0	132.0					
112:	35.0	6.0	10.0	244.0					
113:	45.0	6.0	10.0	185.0					
114:	55.0	6.0	10.0	121.0					
115:	65.0	6.0	10.0	73.0					
116:	75.0	6.0	10.0	263.0					
117:	85.0	6.0	10.0	607.0					
118:	95.0	6.0	10.0	489.0					
119:	105.0	6.0	10.0	746.0					
120:	115.0	6.0	10.0	456.0					
121:	125.0	6.0	10.0	264.0					
122:	begin								
123:	pro ty qp2								
124:	end								

